Faculty of Engineering – Shoubra Engineering and Management of Construction Sites Department Duration : 2 hours	Benha University	Final Exam Course: Mathematics 2 Code: EMP 102 Date: May 30 , 2018

The exam consists of one page   No. of questions: 4   Answer **All** questions   Total Mark: 40

Question 1

(a)Find **y`** from the following: 4

(i) $y = \sinh x . \tanh x - \sinh^{-1} x$ (ii) $y = \sin^{-2} x + \tanh^{-1} x . \sin^{-1} x$

(b)Find the integrals: 6

(i)$\int (2x^3 - 3^x)\, dx$ (ii)$\int (\frac{2}{x+3} + \frac{1}{1-x^2})\, dx$ (iii)$\int \frac{x+2}{x^2-4x-5}\, dx$

(iv)$\int \tan^{-1} x\ dx$ (v) $\int (\sinh x - \cos^2 x)\, dx$ (vi)$\int (\cosh x + \sin^2 2x)\, dx$

Question 2

(a)Find the arc length of the curve : $y = e^x$, x in [0, 2]. 3

(b)If the area of the region between the curve $y = 2^x$, x-axis, x in [1, 3], is rotated about x-axis. Find the volume of the generated solid V_x and its total surface area S. Also, find the area A of this region. 7

Question 3

(a)State the definition of : (i)Line   (ii)Parabola   (iii) Ellipse. 3

(b)Separate the lines : $2x^2 - xy - y^2 = 0$ and then find the angle between them. 2

(c)Write the equation of circle where the points (1, – 2), (3, 2) are ends of diameter. 2

(d)Find the points of intersection of circles :

$x^2 + y^2 - 2x - 2y + 1 = 0,\ x^2 + y^2 - 2x + 2y + 1 = 0.$ 2

Question 4

(a)Find the vertex , focus and sketch the parabola : $y^2 - 12x - 8y + 4 = 0.$ 3

(b)Find center, vertices and sketch the ellipse : $4x^2 + y^2 - 8x + 2y + 1 = 0.$ 3

(c)Find center, vertices and sketch the hyperbola : $x^2 - y^2 - 6x + 10 = 0.$ 3

(d)Determine the type of the curve : $x^2 - 2xy - y^2 - 8y + 4 = 0.$ 2

Good Luck   *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a)(i) $y` = \cosh x \,.\, \tanh x + \sinh x \,.\, \operatorname{sech}^2 x - \frac{1}{\sqrt{1+x^2}}$

(ii) $y` = -2(\sin x)^{-3}.\cos x + \frac{1}{1-x^2}.\sin^{-1} x + \tanh^{-1} x.\frac{1}{\sqrt{1-x^2}}$

--*4- Marks*

(b)(i) $I = \frac{2}{4}x^4 - \frac{3^x}{\ln 3} + c$

(ii) $I = 2\ln(x+3) + \tanh^{-1} x + c$

(iii) $\int \frac{x+2}{x^2-4x-5}\,dx = \int (\frac{\frac{7}{6}}{x-5} - \frac{\frac{1}{6}}{x+1})\,dx = \frac{7}{6}\ln(x-5) - \frac{1}{6}\ln(x+1) + c$

(iv) $\int \tan^{-1} x\,dx = x\ \tan^{-1} x - \int \frac{x}{1+x^2}\,dx = x\ \tan^{-1} x - \frac{1}{2}\ln(1+x^2) + c$

(v) $I = \int (\sinh x - \frac{1}{2}(1+\cos 2x))\,dx = \cosh x - \frac{1}{2}(x + \frac{1}{2}\sin 2x) + c$

(vi) $I = \int (\cosh x + \frac{1}{2}(1-\cos 4x))\,dx = \sinh x + \frac{1}{2}(x - \frac{1}{4}\sin 4x) + c$

--*6- Marks*

Answer of Question 2

(a)The arc length $L = \int_0^2 \sqrt{1+e^{2x}}\,dx = 6.79$

--*3- Marks*

(b)(i) $V_x = \pi \int_1^3 4^x\,dx = 43.28\,\pi$

The total surface area $S = S_x + area\ of\ circle,\ \ radius\ 2 + area\ of\ circle,\ \ radius\ 8.$

$$S_x = 2\pi \int_1^3 2^x.\sqrt{1+(\ln 2)^2.(2^x)^2}\,dx = 62.77\,\pi$$

Then $S = 62.77\,\pi + 4\,\pi + 64\,\pi = 130.77\,\pi$

The aera of the region is : $A = \int_1^3 2^x\,dx = 8.66$

--*7- Marks*

Answer of Question 3

(a)Definitions : Line, Parabola, Ellipse

--*3- Marks*

(b) $2x^2 — xy - y^2 = (2x + y)(x - y) = 0$

The two lines are : 2x + y = 0 , x – y = 0 and $\tan\theta = \frac{-2-1}{1-2} = 3$

--*2- Marks*

(c)The circle is : $(x - 1)(x - 3) + (y + 2)(y - 2) = 0$ Or $x^2 + y^2 - 4x - 1 = 0$

--*2- Marks*

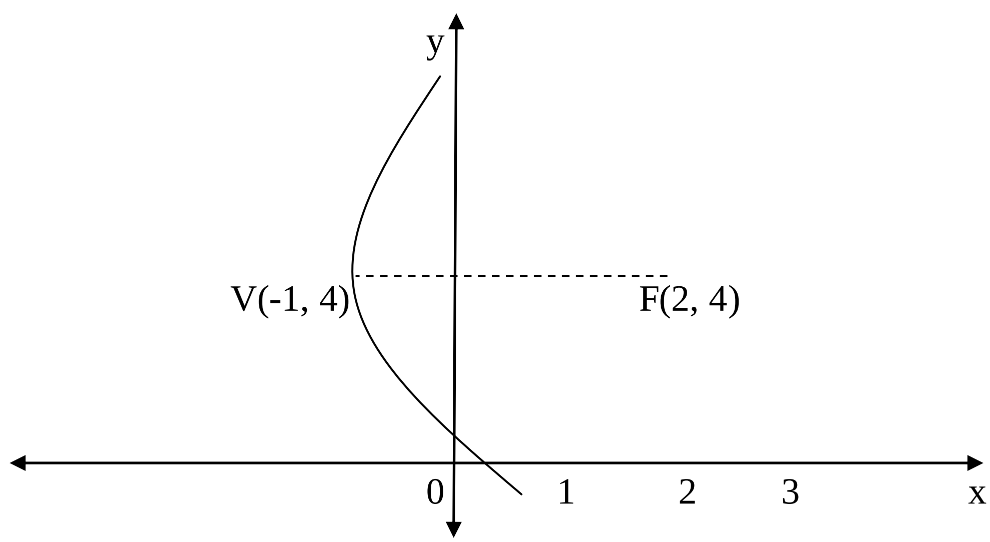
(d)The radical axis is : y = 0. Then from the equation $x^2 - 2x + 1 = 0$, we get x = 1 and the point of intersection is (1, 0).

--*2- Marks*

Answer of Question 4

(a)From : $y^2 - 12x - 8y + 4 = 0$, we get $(y - 4)^2 = 12(x + 1)$

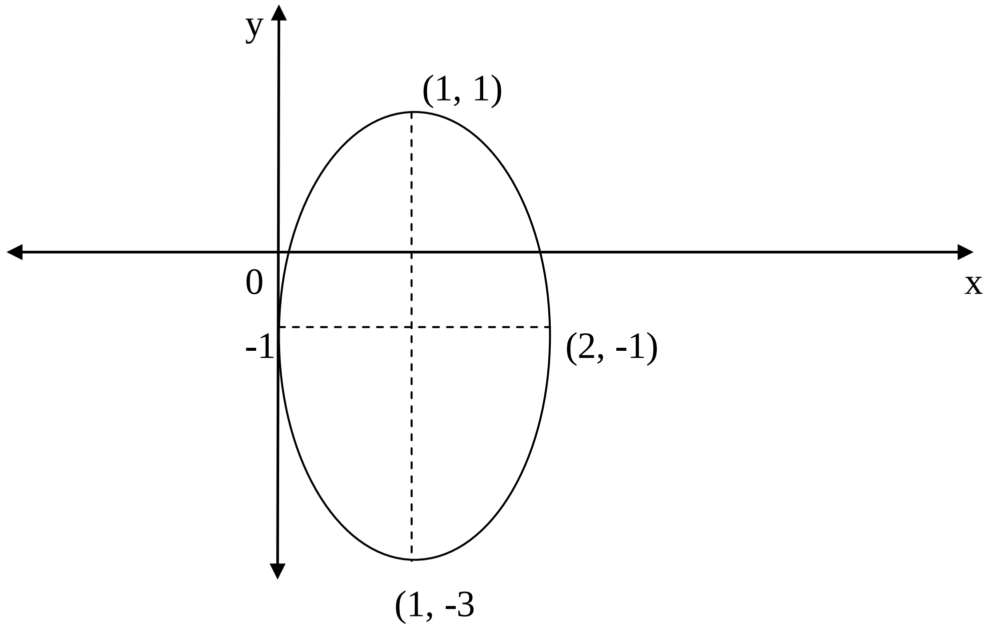
The vertex is (–1, 4) , a = 3, horizontal.



--*3- Marks*

(b)From : $4x^2 + y^2 - 8x + 2y + 1 = 0$, we get $\frac{(x-1)^2}{1} + \frac{(y+1)^2}{4} = 1$

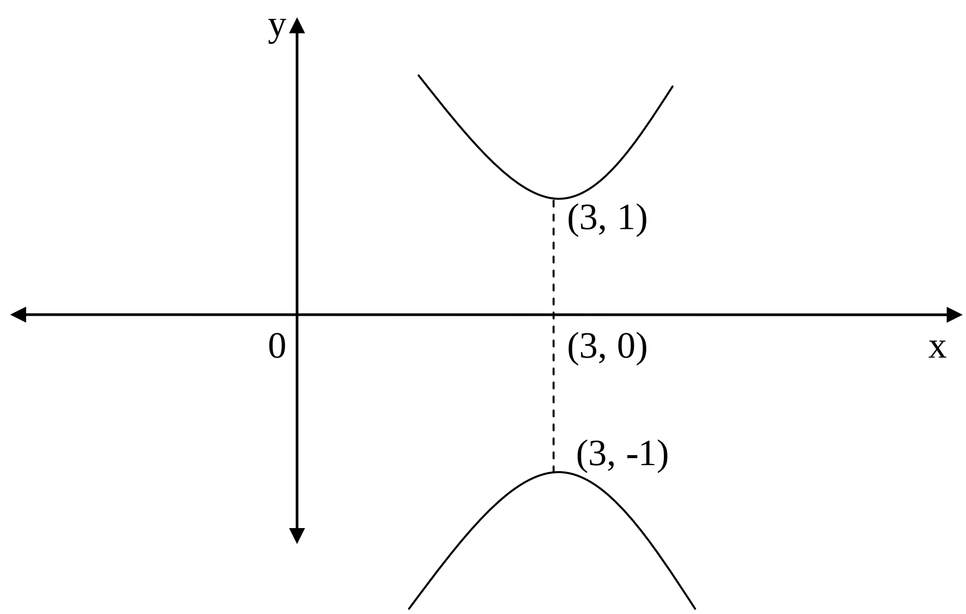
The center is (1, -1) , a = 1, b = 2, vertical.



---*3- Marks*

(c)From : $x^2 - y^2 - 6x + 10 = 0$, we get $y^2 - (x-3)^2 = 1$

The center is (3, 0) , b = 1, vertical.



---*3- Marks*

(d) $\Delta = \begin{vmatrix} 1 & -1 & 0 \\ -1 & -1 & -4 \\ 0 & -4 & 4 \end{vmatrix} = -24$ and $a.b = -1 < h^2 = 1$. Then it is hyperbola.

---*2- Marks*

Dr. Mohamed Eid